



TRAINEE'S NAME
MODULE
CLASS

MATHEMATICS HOLIDAY PACKAGE FOR LEVEL 5 ALL (SOD, NIT & MMP)

INSTRUCTIONS

***The paper is composed by TWO sections;**

***Section I: COMPULSORY**

***Section II: COMPULSORY**

*** EXAM duration: 2 WEEKS**

***Mathematical instruments are allowed.**

Section A : Section 1 Compulsory questions / 55 marks

01. Differentiate $f(x) = (2x^4 + 1)^4$

02. Evaluate the following limits:

a) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} =$

b) $\lim_{x \rightarrow \infty} \frac{7x^4 - 3x^3 + x^2 + 2x + 1}{5x^4 + x^2 - x - 2} =$

03. Given that the function;

$$f(x) = \frac{x^2 + 3}{x - 1} = Ax + B + \frac{C}{x - 1}$$

a. Determine the value of each of the constants A, B, and C

b. Hence, or otherwise, evaluate: $\int \frac{x^2 + 3}{x - 1} dx$ Giving the answer in terms of natural logarithms.

04. a) Convert the following degree into π radians:

i) 135° ii) 225°

b) Convert the following π radians into degrees: $\frac{5\pi}{6}$

05. Evaluate: a) $\int x^2 \ln x dx$ b) $\int x^3 dx$ c) $\int x \cos 8x dx$

06. Evaluate the following integral: a. $\int \frac{7}{(5x+4)^6} dx$ b. $\int (\frac{2t^3+1}{t^4+2t}) dt$

07. Solve the following equations: a) $\begin{cases} 5x + 3y = 12 \\ 7x + 2y = 19 \end{cases}$ b) $16^{2x-5} = 64$

08. Given the complex numbers $z = 3 + i$ and $w = 4 - 3i$

Evaluate: a) i) $3z+w$ ii) $w-2z$ iii) $\left| \frac{z}{w} \right|$

b) Simplify: a) i^{77} b) i^{220}

09. Find $\int x^2(4x^3 + 3)^4 dx$

10. Given that $f(x) = 3 - 7x + 5x^2 - x^3$, show that $3 - x$ is a factor of $f(x)$.
Factorize $f(x)$ completely and hence state the set of values for which $f(x) \geq 0$

11. Calculate for which value(s) of m the equation $x^2 + 3mx + m + \frac{5}{4} = 0$ has:

- a. Two real roots
- b. One double root
- c. No real root

SECTION II: ATTEMPT ONLY 3 QUESTIONS /45 marks

13. a) Solve the following equations: i) $P(z) = 3z^4 - 5z^3 + 5z^2 - 5z + 2$

ii) $\ln(2x + 3) + \ln(-5x + 4) = \ln(-7x + 2)$

b) Express the complex number $z=1+i$ in polar form

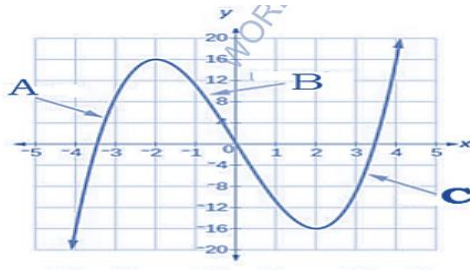
14. i) State whether or not the following functions are even or odd and Justify your answer.

a) $f(x) = x^3 - 3x$

b) $f(x) = x^4 \cos 3x$

c) $f(x) = \frac{x+3}{x-2}$

ii) Consider the function $f(x)$ has the following graph. For the critical points A, B and C state where the function is increasing and decreasing.



15.a) Express $f(x) = \frac{3x^3 - x^2 - 13x - 13}{x^2 - x - 6}$ in partial fractions. Then find antiderivative of $f(x)$.

b) The equation of a curve is $y = 3x^2 - x^3$. the gradient at the point **M** on the curve is 6. Find the equation of the tangent and normal to the curve at **M**.

16. Consider the function defined by $y = x^2 + 1$ and the line $D \equiv x + y = 3$ on interval $[0, 2]$.

- Find the interception points between the line and the curve of the function.
- Draw the function and the line in the same plane.
- Use the integration to calculate the area bounded by two curves.

17. Given that: $f(x) = e^{-2x}$, find:

- The domain of definition,
- All possible asymptotes,
- The interval for which the function is increasing or decreasing;
- The coordinates of its extrema points and state whether they are minimum or maximum;
- Sketch the graph of the given function

18. Given the function: $y = x \ln x$, find:

- a) Find domain of definition;
- b) Calculate limits at boundaries of domain;
- c) Find the first derivative and study signs;
- d) Find the second derivative and the inflection point;
- e) Sketch the graph of the given function above on a).

MATHEMATICS GIVE US HOPE THAT EVERY PROBLEM HAS A SOLUTION!!